EFFECT ON THE RESERVE RATIO OF CHANGES IN RESERVES AND IN LIABILITIES

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With the growing importance of the reserve ratio of Federal Reserve banks as an indicator of the credit condition of the country, it has become of interest to study in detail the various elements that have an effect on that ratio. In this paper no reference is made to the economic and financial conditions affecting the reserve position of the Federal Reserve banks, but an effort is made to determine the effect on the reserve ratio of changes in reserves and in liabilities.

In order to make this discussion understandable to persons not familiar with Federal Reserve banking, it may be stated that the reserve ratio of the Federal Reserve banks is calculated by dividing total cash reserves by combined liabilities on deposits and on Federal Reserve notes. The formula is as follows: Reserve ratio $= \frac{\text{Cash reserves}}{\text{Deposits} + \text{Notes}}$. It is clear, therefore, that the reserve ratio increases as cash reserves increase, and decreases as liabilities increase, and vice versa.

The problem was brought prominently to the attention of the Federal Reserve Board by the large increase in reserves in recent months (owing to the inward gold movement) accompanied by continuous declines in liabilities. The reserve ratio stood at the end of May at above 57 per cent, as compared with a low ratio of 42 per cent about a year ago. It has been a matter of interest and of importance to determine to what extent this increase in the reserve ratio has been caused by additions to reserves, and to what extent by reductions of liabilities.

One way of studying the matter is by a comparison of two given dates, and another way is by an analysis of continuous changes for a given period. Each of these methods lends itself to study in a rough approximate way, but each can also be approached with a view to reducing it to an exact mathematical formula or index. The various methods that have been followed will be briefly described in the discussion below.

COMPARISON FOR TWO GIVEN DATES

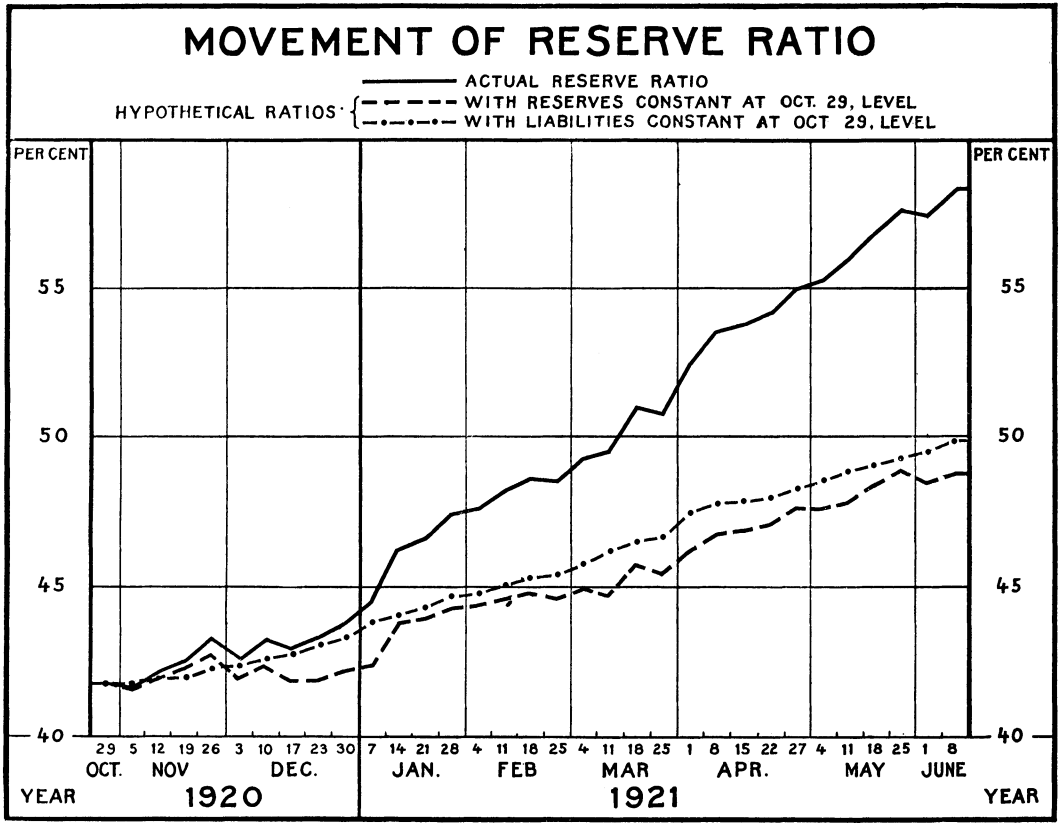
Selecting for the purpose of this discussion the end of October, 1920, and the end of May, 1921, as the boundaries of the period to be studied,

we find the pertinent facts to have been as follows: At the end of October, 1920, the banks' cash reserves were $2,168,038,000, total deposits, $1,845,722,000, and Federal Reserve note circulation, $3,351,-303,000, resulting in a reserve percentage of 41.7. By June 1, 1921, these figures had changed in the following manner: Reserves had increased by $400,787,000, or 18.5 per cent, deposits had declined by $125,332,000, and notes, by $600,004,000, the aggregate decline in liabilities being $725,336,000, or 13.96 per cent. The reserve ratio on the latter date was 57.4 per cent. A rough way of estimating the effect of the two factors, or of the three factors, on the reserve ratio is by assuming consecutively that each of the factors had remained unchanged, and comparing the resultant reserve ratios with the actual reserve ratio. In the present case it would work out as follows: If reserves had remained stationary at the end-of-October figure of $2,168,038,000, the reserve ratio on June 1 would have been 48.5 per cent. If note circulation had remained stationary, the percentage would have been 50.6 per cent; if deposit liabilities had remained unchanged, it would have been 55.9 per cent; if no change had taken place in either class of liabilities, or if the changes had been mutually compensatory, the ratio would have been 49.4 per cent. The situation can be summarized as follows:

	Per cent
Actual ratio on June 1, 1921	57.4
Hypothetical ratio, with the specified items unchanged at October 29, 1920, level:	
Reserves	48.5
Total liabilities	49.4
Deposits	55.9
Notes	50.6

From these figures it appears that changes in reserves were the largest factor in the increase of the reserve ratio between October 29 and June 1, as the retention of the reserves at their October level would have caused the greatest departure from the ratio actually shown for June 1. But reduction in liabilities was nearly as powerful a factor, while of the two kinds of liabilities, notes in circulation affected the ratio a great deal more than deposits, which have changed comparatively little since October.

This method gives a general impression of the changes that took place between the two terminal dates, but an effort was made also to obtain a formula that would distribute the increase in the reserve ratio between the factors involved with mathematical accuracy. Such an algebraic formula was worked out by Mr. J. R. Van Fossen of the Board's staff. He assumed that A equals the percentage increase in reserves and B the percentage increase in combined Federal Reserve


MOVEMENT OF RESERVE RATIO
ACTUAL RESERVE RATIO
HYPOTHETICAL RATIOS
WITH RESERVES CONSTANT AT OCT. 29, LEVEL
WITH LIABILITIES CONSTANT AT OCT 29, LEVEL
PER CENT
55
50
45
40
29 5 12 19 26 3 10 17 23 30 7 14 21 28 4 11 18 25 4 11 18 25 1 8 15 22 27 4 11 18 25 1 8
OCT. NOV DEC. JAN. FEB MAR APR. MAY JUNE
YEAR
1920
1921

note and deposit liabilities. Since the reserve ratio varies directly as reserves and inversely as combined note and deposit liabilities, A will represent also the percentage increase in the reserve ratio, when liabilities remain constant, while $\frac{1}{1+B}-1$ will represent the percentage increase in reserve ratio when reserves remain constant and liabilities alone vary. When both reserves and combined liability vary, the percentage change in the resulting reserve ratio will be caused by the two factors in the ratio of A to $\left(\frac{1}{1+B}-1\right)$ or $\left(\frac{-B}{1+B}\right)$. Accordingly, the effect attributable to the change in reserves is given by the expression $\frac{A}{A+\left(\frac{-B}{1+B}\right)}$ times the percentage increase in the reserve ratio, while that of the liabilities is given by the expression $\frac{\frac{-B}{1+B}}{A+\left(\frac{-B}{1+B}\right)}$ times the percentage increase in the reserve ratio (R) Simplifying these expressions we obtain $R\left(\frac{A+AB}{A+AB-B}\right)$ and $R\left(\frac{-B}{A+AB-B}\right)$.

This formula works out satisfactorily the apportionment of changes in the reserve ratio as long as both of the factors affect the ratio in the same direction, *i. e.*, as long as they move in opposite directions. When, however, reserves and liabilities both increase or both decrease, the formula gives illogical results because of technical mathematical reasons having to do with apportionment of a quantity in proportion to negative and positive factors and with the disturbing appearance of zero in the equation at the point where the two opposite changes exactly offset each other. For this reason it is not possible to use this formula in cases in which reserves have increased and liabilities have also increased. To meet this situation, which apparently does not lend itself to accurate mathematical determination, a method of approximation has been devised. To take a concrete though purely hypothetical example: Suppose that cash reserves equaled 100 and liabilities 200, then the ratio would be $\frac{100}{200}$, or 50 per cent. Suppose that reserves increased by 50 per cent to 150 and liabilities by 12½ per cent to 225, then the ratio would be $\frac{150}{225}=66\frac{2}{3}$ per cent, an increase in ratio from 50 to 66⅔ per cent, or (on the basis of 50 as 100) an increase of 33⅓ per

cent. The problem is: What are the two factors, one positive and one negative, that have resulted in the 33⅓ per cent increase in reserves? The 50 per cent increase in reserves, with liabilities constant, would have caused a 50 per cent increase in ratio; the 12.5 per cent increase in liabilities, with reserves constant, would have caused a 11.3 per cent decrease in the ratio; and the result of the two factors operating independently would have been an increase of 50 per cent and a decrease of 11 per cent, giving an ultimate increase of 39 per cent. But the actual increase was only 33 per cent. Distribute the difference $(39-33=6)$ in proportion to the "motive power" of the changes in reserves and in liabilities, namely 50 and 11, or roughly 5 to 1. The increase due to the reserves will thus not be 50 per cent, but only 45 per cent, and the decrease due to increase in liabilities will not be 11 per cent, but 12 per cent. Answer: the increase of 33 per cent in ratio is attributable roughly to an increase of 45 per cent due to the increase in reserves, offset in part by a decrease of 12 per cent caused by the increase in liabilities. This method, although not absolutely accurate, affords a sufficiently close approximation to accuracy to answer all practical requirements.

The apportionment between notes and deposits of the increase or decrease in ratio attributable to the change in liabilities is a simple problem in proportion. The division must be in proportion to the effect that the change in each factor has had on the combined liabilities. Further details on this simple point do not appear to be necessary in this discussion.

CONTINUOUS COMPARISONS FOR A PERIOD OF TIME

As against the above methods of comparing two separate dates, there have been made also studies of the week-to-week relationship between the changes in reserves and in liabilities. Here again a crude general study may be made by assuming that each factor in turn remained constant, and in seeing how the result in the reserve ratio would vary from the actual reserve ratio. The accompanying chart is based on figures computed on these assumptions. It shows that on the whole changes in reserves and in liabilities have been of approximately equal importance in causing changes in the reserve ratio during the period under discussion, although the changes in reserves have been slightly more potent than changes in liabilities, as evidenced by the fact that the curve showing the ratio with reserves constant is almost continuously lower than the curve showing the ratio with liabilities constant.

However, a more accurate way of determining these relationships was thought desirable, and a coefficient of correlation was worked out. But the results were unsatisfactory, mainly because the coefficient of

correlation in its derivation assumes a norm or base and works from that base. The Pearson coefficient originated in biological study where a norm is assumed to exist. In banking statistics during a period of rapid change without much historic precedent, there is no norm, and a correlation worked out in relation to an arbitrary amount for a given date or to a short-time average is not a satisfactory method of procedure. For this reason it was decided to compare the changes from week to week, using each successive week as a base for the change for the following week. In this manner an index of divergence was worked out. In developing the index, the successive steps may be described as follows:

1. The actual amounts of each of the five items involved (reserves, notes, deposits, combined liabilities, and ratio) were listed in millions of dollars for each week from October 29, 1920, to June 1, 1921.

2. For each of the items, the increases or decreases from one week to the next in absolute amounts were written in a second column.

3. The percentages of increase or decrease for each week were calculated.

4. For each set of quantities the divergence between the percentage of change from week to week was figured.

5. These divergences were added and then divided by the number of weeks; the quotient is "the index of divergence."

Of the various steps in the procedure, only the fourth calls for discussion, the others being plain arithmetical propositions.

In the following table the absolute amounts are omitted in order to save space and also because they have no bearing on the method; only the percentages and the divergences are shown. In order to understand the method, it should be borne in mind that what is done is to compare the four sets of relationships:

(*a*) Between changes in the reserve ratio and in cash reserves.

(*b*) Between changes in the reserve ratio and in deposit and note liabilities combined.

(*c*) Between changes in the reserve ratio and in Federal Reserve notes.

(*d*) Between changes in the reserve ratio and in net deposits.

To begin with the first relationship:

During the week ending November 5, cash reserves increased by 0.08 per cent. During the same week, the reserve ratio decreased by 0.10 per cent. The relationship between cash reserves and the reserve ratio being direct, a decline in cash reserves—other things being equal—would result in a proportionate decline in the ratio. The divergence, therefore, equals the sum of the percentage of increase in

Week ending—	Percentage of increase (+) or of decrease (—) in					Divergence in percentage changes in reserve ratio and—			
	Cash reserves	Federal Reserve notes	Total deposits	Notes and deposits combined	Reserve ratio	Cash reserves	Federal Reserve notes	Total deposits	Notes and deposits combined
1920									
October 29	+ .50	— .15	+1.59	+ .47	+ .02	.48	.13	1.61	.49
November 5	+ .08	+ .09	+ .32	+ .17	— .10	.18	.01	.22	.07
12	+ .47	— .75	— .33	— .60	+1.06	.59	.31	.73	.46
19	+ .01	— .65	—1.36	— .90	+ .95	.94	.30	.41	.05
26	+ .69	+ .55	—3.83	—1.01	+1.72	1.03	2.27	2.11	.71
December 3	+ .13	— .41	+5.69	+1.70	—1.55	1.68	1.96	4.14	.15
10	+ .65	— .01	—2.07	— .75	+1.41	.76	1.40	.66	.66
17	+ .45	+ .98	+1.03	+1.00	— .53	.98	.45	.50	.47
23	+ .64	+1.81	—3.24	+ .02	+ .61	.03	2.42	2.63	.63
30	+ .55	—1.77	+1.56	— .63	+1.20	.65	.57	2.76	.57
1921									
January 7	+1.23	—2.23	+2.67	— .52	+1.76	.53	.47	4.43	1.24
14	+ .51	—3.38	—2.93	—3.22	+3.84	3.33	.46	.91	.62
21	+ .55	—1.40	+1.69	— .28	+ .84	.29	.56	2.53	.56
28	+ .81	— .79	— .82	— .80	+1.63	.82	.84	.81	.83
February 4	+ .27	— .49	+ .52	— .12	+ .38	.11	.11	.90	.26
11	+ .69	— .82	— .16	— .58	+1.26	.57	.44	1.10	.68
18	+ .46	— .43	— .42	— .42	+ .89	.43	.46	.47	.47
25	+ .17	+ .47	+ .12	+ .34	— .15	.32	.32	.03	.19
March 4	+ .80	— .30	—1.26	— .66	+1.46	.66	1.16	.20	.80
11	+ .93	—1.21	+3.21	+ .43	+ .51	.42	.70	3.72	.94
18	+ .70	—1.43	—3.75	—2.31	+3.07	2.37	1.64	.68	.76
25	+ .30	—1.09	+3.73	+ .72	— .41	.71	1.50	3.32	.31
April 1	+1.62	— .77	—2.81	—1.56	+3.23	1.61	2.46	.42	1.67
8	+ .84	— .49	—2.45	—1.24	+2.10	1.26	1.61	.35	.86
15	+ .13	— .88	+ .55	— .34	+ .47	.34	.41	1.02	.13
22	+ .31	— .41	— .32	— .38	+ .69	.38	.28	.37	.31
27	+ .48	— .93	— .13	—1.09	+1.59	1.11	.66	1.46	.50
May 4	+ .60	— .05	+ .18	+ .03	+ .56	.04	.51	.74	.59
11	+ .71	— .84	+ .26	— .42	+1.14	.43	.30	1.40	.72
18	+ .45	—1.34	— .97	—1.20	+1.66	1.21	.32	.69	.46
25	+ .36	—1.18	— .62	— .97	+1.34	.98	.16	.72	.37
June 1	+ .41	+ .60	+ .85	+ .70	— .28	.69	.32	.57	.42
Total divergence						25.93	25.51	42.61	17.95
Index of divergence						.81	.80	1.33	.56

cash reserves and the percentage of decrease in the reserve ratio, that is, $0.08+0.10=0.18$. The same calculation is made for each week of the period. These divergences are then added, totaling 25.45, which, when divided by the number of weeks, gives an index of divergence of 0.82.

In measuring the relationship between the reserve ratio and the liabilities, either separately or combined, the method of procedure is modified by the fact that the relationship is inverse. For example, taking the same week as above, Federal Reserve notes and deposits combined show an increase of 0.17 per cent. During the same week, the reserve ratio shows a decrease of 0.10 per cent. The difference between these declines is 0.07. The same calculation for each week of the period, when averaged, gives an index of divergence of 0.56.

The method used in the calculation of the index of divergence of Federal Reserve notes and total deposits is exactly the same as that used in the calculation of the two combined. The indices of divergence are 0.80 for Federal Reserve notes and 1.33 for total deposits. In view of the fact that Federal Reserve notes and total deposits are each only part of the divisor in calculating the ratio, it is to be expected that the divergence would be greater in the case of those partial factors than in the case of total cash reserves or of total liabilities. It will be noted, however, that the index of divergence for Federal Reserve notes is no larger than that for cash reserves. This means that in spite of the handicap of being only a partial factor, Federal Reserve notes have, as a matter of fact, fluctuated in as close harmony with the reserve ratio as have cash reserves. On the other hand, total deposits show an index of divergence that is much greater than that of any of the other factors involved.

It is admitted that this method is crude. A much more refined way of arriving at the result has been worked out by G. P. Watkins in a paper to be published later. Watkins works with logarithms instead of percentages, because logarithms can be added, subtracted, and divided without damage to their correctness, whereas percentages are not adapted to that use for accurate purposes for the reason that an increase of 10 per cent does not correspond to a decrease of 10 per cent, but to one of 9 per cent. Watkins, who attacked the problem in connection with a study of grain prices, also makes corrections for variability and trend, and his results are of a high order of mathematical accuracy. For the purposes of the Federal Reserve Board, however, it is probable that the index of divergence, as described in this paper, is sufficiently accurate to indicate the relative closeness of variations in the reserve ratio and in reserves, on the one hand, and in notes and deposits on the other.